Derivations built by `mjc`

Warning

Multisets are represented as sets (formulas are not duplicated) and dots denote empty multisets; $\neg A$ abbreviates $A \supset \bot$.

Formula label: `tertium`

$$p \vee \neg p$$

Derivation in MJcr

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\langle \cdot \rangle \cdot p \to p \vdash \bot}\,\mathrm{Ax}}{p \to \bot \vdash p}\,\mathrm{Bot}\,\mathrm{Ctr}}{\cdot \to \neg p \vdash p}\,\mathrm{Bot}}{\cdot \to p \vee \neg p}\,\mathrm{Ctr}}{\cdot \to p \vee \neg p \vdash \neg p}\,\mathrm{Ctr}$$

Translation in MJc

$$\dfrac{\dfrac{\dfrac{}{p,\, p^{\perp\perp},\, \perp^{\perp\perp},\, \neg p^{\perp},\, (p \vee \neg p)^{\perp\perp} \vdash p}\,\mathrm{Ax}}{p,\, p^{\perp\perp},\, \perp^{\perp\perp},\, \neg p^{\perp},\, (p \vee \neg p)^{\perp\perp} \Rightarrow p}\,\mathrm{LG}_{\perp}}{p,\, p^{\perp\perp},\, \neg p^{\perp},\, (p \vee \neg p)^{\perp\perp} \Rightarrow \perp}\,\mathrm{RC}$$
$$\dfrac{p^{\perp\perp},\, \neg p^{\perp},\, (p \vee \neg p)^{\perp\perp} \Rightarrow \neg p}{\neg p^{\perp},\, (p \vee \neg p)^{\perp\perp} \Rightarrow p}\,\mathrm{RC}$$
$$\vdash \Rightarrow p \vee \neg p\,\mathrm{RV}^{\vdash}$$

The rule

$$\dfrac{(A \vee B)^{\perp\perp}, B^{\perp}, \Gamma \Rightarrow A}{\Gamma \Rightarrow A \vee B}\,\mathrm{RV}^{\vdash}$$

can be derived in **MJc** in few steps.

Translation in NC

$$\dfrac{\dfrac{\dfrac{}{p,\,p^{\perp},\,\perp^{+},\,\neg p^{+},\,(p\vee\neg p)^{+}\rhd p_{\Sigma}^{+}}\,\mathrm{id}}{p,\,p^{\perp},\,\perp^{+},\,\neg p^{+},\,(p\vee\neg p)^{+}\rhd p_{\Sigma}^{\perp}}\,\perp_{\Sigma}\mathcal{E}}{p,\,p^{\perp},\,\neg p^{+},\,(p\vee\neg p)^{+}\rhd\perp_{\Sigma}^{+}}\,\perp_{\Sigma}\mathcal{I}}{\dfrac{p^{\perp},\,\neg p^{+},\,(p\vee\neg p)^{+}\rhd\neg p_{\Sigma}^{+}}{\neg p^{+},\,(p\vee\neg p)^{+}\rhd p_{\Sigma}^{+}}\,\neg_{\Sigma}\mathcal{E}}\,\neg_{\Sigma}\mathcal{I}}{\vdash p\vee\neg p^{\perp}}\,\vee^{+}_{\Sigma}\mathcal{I}$$

The rule

$$\dfrac{(A\vee B)^{\perp},\,B^{\perp},\,\Gamma\rhd A^{\perp}}{\Gamma\vdash A\vee B^{\perp\!\perp}}\,\vee^{+}_{I}$$

can be derived in **NC** in few steps.

Derivations built by `mjc`

Warning
Multisets are represented as sets (formulas are not duplicated) and dots denote empty multisets; $\neg A$ abbreviates $A \supset \bot$.

Formula label: dummett

$$(p \supset q) \vee (q \supset p)$$

Derivation in MJcr

$$
\begin{array}{lcl}
A_0 & = & p \supset q \\
A_1 & = & q \supset p \\
D & = & A_0 \vee A_1 \quad = \quad (p \supset q) \vee (q \supset p)
\end{array}
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma_0', p \supset q \Rightarrow p \supset q}\,\mathrm{Ax}}{\Gamma_0' \Rightarrow p \supset q, \neg(p \supset q)}\,\mathrm{L}\neg}{p, q \Rightarrow p \supset q}\,\mathrm{R}\supset}{p \Rightarrow q, A_1}\,\mathrm{R}\supset}{\Rightarrow A_0, A_1}\,\mathrm{R}\vee
$$

$$
\Rightarrow D
$$

Translation in MJc

$$
\begin{array}{rcl}
A_0 & = & p \supset q \\
A_1 & = & q \supset p \\
D & = & A_0 \vee A_1 \quad = \quad (p \supset q) \vee (q \supset p)
\end{array}
$$

$$
\begin{array}{c}
\overline{p, q, p^{\circ}, q^{\circ}, A_1^{\circ}, D^{\circ} \Rightarrow p \rightarrow q} \; {}_{\mathrm{IC}_0} \\
\overline{p, q, p^{\circ}, q^{\circ}, A_1^{\circ}, D^{\circ} \Rightarrow p} \; {}_{\mathrm{RC}} \\
\overline{p, q, q^{\circ}, A_1^{\circ}, D^{\circ} \Rightarrow p} \; {}_{\mathrm{RC}} \\
\overline{p, q^{\circ}, A_1^{\circ}, D^{\circ} \Rightarrow A_1} \; {}_{\mathrm{RC}} \\
\overline{p, A_1^{\circ}, D^{\circ} \Rightarrow A_1} \; {}_{\mathrm{RC}} \\
\overline{A_1^{\circ}, D^{\circ} \Rightarrow A_1} \; {}_{\mathrm{RC}} \\
\overline{\cdot \Rightarrow D}
\end{array}
$$

The rule

$$
\frac{(A \vee B)^{\circ}, D^{\circ}, \Gamma \Rightarrow A}{\Gamma \Rightarrow A \vee B} \; {}_{\mathrm{RC}},
$$

can be derived in **MJc** in few steps.

Translation in NC

$$
\begin{array}{rcl}
A_0 & = & p \supset q \\
A_1 & = & q \supset p \\
D & = & A_0 \vee A_1 \quad = \quad (p \supset q) \vee (q \supset p)
\end{array}
$$

$$
\begin{array}{c}
\underline{p, q, p^\top, q^\top, A_1^\top, D^\top \vdash p_1^\top} \\
\underline{p, q, p^\top, q^\top, A_1^\top, D^\top \vdash p_2^\top} \\
\underline{p, q, q^\top, A_1^\top, D^\top \vdash p_1^\top} \\
\underline{p, q^\top, A_1^\top, D^\top \vdash A_1^\top} \\
\underline{p, A_1^\top, D^\top \vdash q_1^\top} \\
\underline{A_1^\top, D^\top \vdash A_0^\top} \\
\vdash D^\top
\end{array}
$$

The rule

$$
\frac{(A \vee B)^\top, B^\top, \Gamma \vdash A^\top}{\Gamma \vdash A \vee B^\top} \vee^\top_l
$$

can be derived in **NC** in few steps.

Derivations built by `mjc`

Warning

Multisets are represented as sets (formulas are not duplicated) and dots denote empty multisets; $\neg A$ abbreviates $A \supset \bot$.

Formula label: doubleSeg

$$\neg p \supset p$$

Derivation in MJcr

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\langle\cdot|p \to p\rangle \bot}\,\mathrm{Ax}}{p \to \bot \mid p}\,\mathrm{Ed}\,\mathrm{Ef}}{\cdot \Rightarrow \neg p \mid p}\,\mathrm{Rc}\supset \qquad \dfrac{}{\langle\cdot|\bot \to \bot\rangle \bot}\,\mathrm{L}\bot}{\langle\cdot|\neg p \to \bot\rangle \bot}\,\mathrm{L}\bot}{\neg p \Rightarrow p \mid \bot}\,\mathrm{Rc}\supset}{\cdot \Rightarrow \neg p \supset p \mid \bot}\,\mathrm{Rc}\supset$$

Translation in MJc

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{p, p^{\bot}, \bot^{\bot}, \neg p \vdash p}\,\mathrm{ax}}{p, p^{\bot}, \bot^{\bot}, \neg p \vdash p}\,\mathrm{MC_1}}{p, p^{\bot}, \neg p \vdash \bot}\,\mathrm{ec}_1 \qquad \dfrac{}{p^{\bot}, \neg p, \bot \vdash p}\,\mathrm{i}_1}{p^{\bot}, \neg p \vdash \neg p}\,\mathrm{i}_2}{\dfrac{p^{\bot}, \neg p \vdash \neg p \to p}{\neg p \vdash \neg p \supset p}\,\mathrm{ec}}\,\mathrm{MC_1}$$

Translation in NC

$$\dfrac{\dfrac{}{p, p^{\bullet}, \bot^{\bullet}, \neg\!\!\rightarrow p \vdash p^{\bullet}}\,\mathrm{id}}{p, p^{\bullet}, \bot^{\bullet}, \neg\!\!\rightarrow p \vdash \bot^{\bullet}}\,\mathrm{id}}{p, p^{\bullet}, \neg\!\!\rightarrow p \vdash \bot^{\bullet}}\,\mathrm{id}\,\mathrm{E}$$

$$\dfrac{\dfrac{\dfrac{}{p^{\bullet}, \neg\!\!\rightarrow p \vdash \neg\!\!\rightarrow p^{\bullet}}\,\mathrm{id}}{p^{\bullet}, \neg\!\!\rightarrow p \vdash \neg\!\!\rightarrow p^{\bullet}}\,\mathrm{id}}{p^{\bullet}, \neg\!\!\rightarrow p \vdash \neg\!\!\rightarrow p^{\bullet}}\,\mathrm{id}}{p^{\bullet}, \neg\!\!\rightarrow p \vdash \bot^{\bullet}}\,\mathrm{id}\,\mathrm{E}$$

$$\dfrac{\dfrac{\dfrac{}{p^{\bullet}, \neg\!\!\rightarrow p \vdash \bot^{\bullet}}\,\mathrm{id}}{\neg\!\!\rightarrow p \vdash p^{\bullet}}\,\mathrm{id}}{\neg\!\!\rightarrow p \vdash \neg\!\!\rightarrow p^{\bullet}}\,\mathrm{id}}{\neg\!\!\rightarrow p \vdash \neg\!\!\rightarrow p^{\bullet}}\,\mathrm{id}$$

Derivations built by `mjc`

Warning

Multisets are represented as sets (formulas are not duplicated) and dots denote empty multisets; $\neg A$ abbreviates $A \supset \bot$.

Formula label: `peirce`

$$((p \supset q) \supset p) \supset p$$

Derivation in MJcr

$$A \quad = \quad (p \supset q) \supset p$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\subset (p \to p) \subset \mathcal{R}}\,\mathrm{Ax}}{p \to q \subset p}\,\mathrm{Bot}\,\mathcal{L}}{\cdot \to p \supset q \subset p}\,\mathrm{Ri} \qquad \dfrac{}{\subset (p \to p) \subset \mathcal{L}}\,\mathrm{Ax}}{\dfrac{\subset (A \to p) \subset \mathcal{L}}{A \to p \subset \mathcal{L}}\,\mathrm{Ri}}\,\mathrm{Ri}}{\cdot \to A \supset p \subset \mathcal{L}}\,\mathrm{Ri}$$

Translation in MJc

$$
\begin{array}{c}
A \;=\; (p \supset q) \supset p \\
\hline
p, p^{\circ}, q^{\circ}, A \Rightarrow p \;\; \text{ic}_{\text{ic}} \\
\hline
p, p^{\circ}, q^{\circ}, A \Rightarrow p \;\; \text{ic}_{\text{ic}} \\
\hline
p, p^{\circ}, A \Rightarrow q \;\; \text{ic}_{\text{ic}} \\
\hline
p^{\circ}, A \Rightarrow p \supset q \;\; \text{ic}_{\text{ic}} \qquad \overline{p^{\circ}, A \Rightarrow p \;\; \text{ic}_{\text{ic}}} \\
\hline
p^{\circ}, A, A \Rightarrow p \;\; \text{ic}_{\text{ic}} \\
\hline
p^{\circ}, A \Rightarrow p \;\; \text{ic}_{\text{ic}} \\
\hline
A \Rightarrow p \;\; \text{ic}_{\text{ic}} \\
\hline
\Rightarrow A \supset p \;\; \text{ic}_{\text{ic}}
\end{array}
$$

Translation in NC

$$A = (p \supset q) \supset p$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{p, p^{\perp}, q^{\perp}, A \vdash p, \perp}\,\mathrm{id}}{p, p^{\perp}, q^{\perp}, A \vdash p^{\perp}}\,\neg_{\mathcal{L}}}{\dfrac{}{p, p^{\perp}, A \vdash q^{\perp}}\,\neg_{\mathcal{L}}}}{\dfrac{}{p^{\perp}, A \vdash A, \perp}\,\mathrm{id}} \quad p^{\perp}, A \vdash p \supset q^{\perp}}{\dfrac{\dfrac{p^{\perp}, A \vdash p, \perp}{p^{\perp}, A \vdash p^{\perp}}\,\neg_{\mathcal{R}}}{A \vdash p^{\perp}}\,\neg_{\mathcal{L}}}\,\supset_{\mathcal{L}}$$

$$\cdot \vdash A \supset p^{\perp}\,\neg_{\mathcal{R}}$$

Derivations built by `mjc`

Warning

Multisets are represented as sets (formulas are not duplicated) and dots denote empty multisets; $\neg A$ abbreviates $A \supset \bot$.

Formula label: `deMorg`

$$\neg(p \wedge q) \supset (\neg p \vee \neg q)$$

Derivation in MJcr

$$\begin{array}{lcl} A & = & \neg(p \wedge q) \\ B & = & \neg p \vee \neg q \end{array}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\cdot;\cdot;p \to_0 p;\bot,\neg q}\,\text{Ax}}{p \Rightarrow p;\bot,\neg q}\,\text{Lf}_0 \quad \dfrac{\dfrac{\dfrac{\dfrac{}{p;\cdot;q \to_0 q;\bot}\,\text{Ax}}{p, q \Rightarrow \bot;q,\bot}\,\text{ReLf}}{p \Rightarrow \neg q;q,\bot}\,R\supset}{p \Rightarrow q;\bot,\neg q}\,\text{Rest}}{p \Rightarrow p \wedge q;\bot,\neg q}\,R\wedge \quad \dfrac{}{p;\cdot;\bot \to_1 \bot;\neg q}\,L\bot}{p;\cdot;A \to_1 \bot;\neg q}\,L\supset}{p, A \Rightarrow \bot;\neg q}\,\text{Lf}_1}{A \Rightarrow \neg p;\neg q}\,R\supset}{A \Rightarrow B;\cdot}\,R\vee}{\cdot \Rightarrow A \supset B;\cdot}\,R\supset$$

Translation in MJc

$$
\begin{aligned}
A &= \neg(p \wedge q) \\
B &= \neg p \vee \neg q
\end{aligned}
$$

$$
\dfrac{\dfrac{}{p, q, q^{\perp}, \perp^{\perp}, \neg q^{\perp}, B^{\perp}, A, q \to q}\,\mathrm{Ax}}{p, q, q^{\perp}, \perp^{\perp}, \neg q^{\perp}, B^{\perp}, A \Rightarrow q}\,\mathrm{LC_1}
\qquad
\dfrac{\dfrac{}{p, q, q^{\perp}, \perp^{\perp}, \neg q^{\perp}, B^{\perp}, A \Rightarrow \perp}}{p, q, q^{\perp}, \perp^{\perp}, \neg q^{\perp}, B^{\perp}, A \Rightarrow q}\,\mathrm{RC}
$$

$$
\dfrac{\dfrac{}{p, \perp^{\perp}, \neg q^{\perp}, B^{\perp}, A \Rightarrow p}\,\mathrm{Ax}}{p, \perp^{\perp}, \neg q^{\perp}, B^{\perp}, A \Rightarrow p}\,\mathrm{LC_1}
\qquad
\dfrac{\dfrac{}{p, \perp^{\perp}, \neg q^{\perp}, B^{\perp}, A \Rightarrow \perp}}{p, \perp^{\perp}, \neg q^{\perp}, B^{\perp}, A \Rightarrow q}\,\mathrm{RC}
$$

$$
\dfrac{\dfrac{}{p, \perp^{\perp}, \neg q^{\perp}, B^{\perp}, A \Rightarrow p \wedge q}\,\mathrm{Ax}}{p, \perp^{\perp}, \neg q^{\perp}, B^{\perp}, A \Rightarrow q}\,\mathrm{R_1}
\qquad
\dfrac{\dfrac{}{p, \perp^{\perp}, \neg q^{\perp}, B^{\perp}, A, \perp \to \perp}}{p, \perp^{\perp}, \neg q^{\perp}, B^{\perp}, A \Rightarrow \perp}\,\mathrm{L_\perp}
$$

$$
\dfrac{\dfrac{\dfrac{}{p, \perp^{\perp}, \neg q^{\perp}, B^{\perp}, A, \neg q \to \perp}}{p, \neg q^{\perp}, B^{\perp}, A \Rightarrow \perp}\,\mathrm{LC_1}}{\neg q^{\perp}, B^{\perp}, A \Rightarrow \neg p}\,\mathrm{R_\supset}
$$

$$
\dfrac{\neg q^{\perp}, B^{\perp}, A \Rightarrow \neg p}{\neg \Rightarrow A \supset B}\,\mathrm{R_\supset}
$$

The rule

$$
\dfrac{(A \vee B)^{\perp}, B^{\perp}, \Gamma \Rightarrow A}{\Gamma \Rightarrow A \vee B}\,\mathrm{R_\vee}^{\perp}
$$

can be derived in **MJc** in few steps.

Translation in NC

$$
\begin{array}{lcl}
A & = & \neg(p \wedge q) \\
B & = & \neg p \vee \neg q
\end{array}
$$

$$
\dfrac{\dfrac{\dfrac{}{p,\, p^{\perp},\, \perp^{\perp},\, \neg q^{\perp},\, B^{\perp},\, A^{\perp} \vdash p}\,{\scriptstyle id}}{p,\, p^{\perp},\, \perp^{\perp},\, \neg q^{\perp},\, B^{\perp},\, A^{\perp} \vdash p \wedge q^{\perp}}\,{\scriptstyle \wedge I}}{p,\, p^{\perp},\, \perp^{\perp},\, \neg q^{\perp},\, B^{\perp},\, A^{\perp} \vdash p^{\perp}}\,{\scriptstyle \neg E}}{p,\, \perp^{\perp},\, \neg q^{\perp},\, B^{\perp},\, A^{\perp} \vdash p^{\perp}}\,{\scriptstyle \perp E}}{p,\, \perp^{\perp},\, \neg q^{\perp},\, B^{\perp},\, A^{\perp} \vdash A^{\perp}}\,{\scriptstyle \wedge I}
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{}{p,\, q,\, q^{\perp},\, \perp^{\perp},\, \neg q^{\perp},\, B^{\perp},\, A^{\perp} \vdash q}\,{\scriptstyle id}}{p,\, q,\, q^{\perp},\, \perp^{\perp},\, \neg q^{\perp},\, B^{\perp},\, A^{\perp} \vdash q^{\perp}}\,{\scriptstyle \neg E}}{p,\, q,\, q^{\perp},\, \perp^{\perp},\, \neg q^{\perp},\, B^{\perp},\, A^{\perp} \vdash \perp^{\perp}}\,{\scriptstyle \perp E}}{p,\, q^{\perp},\, \perp^{\perp},\, \neg q^{\perp},\, B^{\perp},\, A^{\perp} \vdash \neg q^{\perp}}\,{\scriptstyle \neg I}}{p,\, \perp^{\perp},\, \neg q^{\perp},\, B^{\perp},\, A^{\perp} \vdash q^{\perp}}\,{\scriptstyle \perp E}
$$

$$
\dfrac{p,\, \perp^{\perp},\, \neg q^{\perp},\, B^{\perp},\, A^{\perp} \vdash A^{\perp}_{\perp} \qquad p,\, \perp^{\perp},\, \neg q^{\perp},\, B^{\perp},\, A^{\perp} \vdash p \wedge q^{\perp}}{p,\, \perp^{\perp},\, \neg q^{\perp},\, B^{\perp},\, A^{\perp} \vdash \perp^{\perp}}\,{\scriptstyle \perp E}
$$

$$
\dfrac{p,\, \neg q^{\perp},\, B^{\perp},\, A^{\perp} \vdash \perp^{\perp}}{\neg q^{\perp},\, B^{\perp},\, A^{\perp} \vdash p^{\perp}}\,{\scriptstyle \neg I}
$$

$$
\dfrac{\neg q^{\perp},\, B^{\perp},\, A^{\perp} \vdash p^{\perp}}{A^{\perp} \vdash B^{\perp}}\,{\scriptstyle \neg I}
$$

$$
\neg\, A \supseteq B^{\perp}\,{\scriptstyle \neg I}
$$

The rule

$$
\dfrac{(A \vee B)^{\perp},\, B^{\perp},\, \Gamma \vdash A^{\perp}}{\Gamma \vdash A \vee B^{\perp}}\,{\scriptstyle \vee I}
$$

can be derived in **NC** in few steps.

Derivations built by `mjc`

Warning
Multisets are represented as sets (formulas are not duplicated) and dots denote empty multisets; $\neg A$ abbreviates $A \supset \bot$.

Formula label: ex4pp8

$$(((p \supset q) \supset p) \wedge (p \supset r)) \supset r$$

Derivation in MJcr

$$\begin{array}{lcll} A & = & B_0 \wedge B_1 & = \quad ((p \supset q) \supset p) \wedge (p \supset r) \\ B_0 & = & (p \supset q) \supset p & \\ B_1 & = & p \supset r & \end{array}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\neg\neg p \to_0 p, r_0}\,Ax}{\neg\neg p \to_0 p, r}\,Ax_t}{\neg\neg p \supset q, p, r}\,R\supset \qquad \dfrac{}{\neg\neg p \to_0 p, r}\,Ax}{\neg\neg p \supset q, r}\,L\supset$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{B_0 \to p, r}\,Ax}{\neg B_0 \to_0 p, r}\,Ax_t \qquad \dfrac{}{\neg B_0, r \to_0 r, \cdot}\,Ax}{\neg B_0, B_0 \to_0 r, \cdot}\,L\supset}{\neg \cdot, A \to_0 r, \cdot}\,L\wedge_1}{A \to r, \cdot}\,Ax_t$$

$$\dfrac{A \to r, \cdot}{\neg\neg A \supset r, \cdot}\,R\supset$$

Translation in MJc

$$
\begin{array}{rcl}
A & = & B_0 \wedge B_1 \quad = \quad ((p \supset q) \supset p) \wedge (p \supset r) \\
B_0 & = & (p \supset q) \supset p \\
B_1 & = & p \supset r
\end{array}
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{p, p^{\circ}, q^{\circ}, r^{\circ}, B_0, p \rightsquigarrow p}\,\mathrm{Ax}}{p, p^{\circ}, q^{\circ}, r^{\circ}, B_0 \Rightarrow p}\,\mathrm{LC_1}}{p, p^{\circ}, r^{\circ}, B_0 \Rightarrow q}\,\mathrm{RC} \qquad \dfrac{}{p^{\circ}, r^{\circ}, B_0, p \rightsquigarrow p}\,\mathrm{Ax}}{p^{\circ}, r^{\circ}, B_0, B_0 \rightsquigarrow p}\,\mathrm{LC_1}}{p^{\circ}, r^{\circ}, B_0 \Rightarrow p}\,\mathrm{RC}}{\dfrac{\dfrac{\dfrac{}{r^{\circ}, A, B_0 \rightsquigarrow}\,\mathrm{Ax}}{r^{\circ}, A, A \rightsquigarrow}\,\mathrm{LC_4}}{r^{\circ}, A \Rightarrow p} \qquad \dfrac{}{r^{\circ}, A, r \rightsquigarrow r}\,\mathrm{Ax}}{\dfrac{\dfrac{\dfrac{r^{\circ}, A, B_1 \rightsquigarrow r}{r^{\circ}, A, A \rightsquigarrow r}\,\mathrm{LC_4}}{r^{\circ}, A \Rightarrow r}\,\mathrm{RC}}{\Rightarrow A \supset r}\,\mathrm{R\supset}}\,\mathrm{L\supset}
$$

Elimination of substitutions

$$\begin{array}{lcll} A & = & B_0 \wedge B_1 & = \quad ((p \supset q) \supset p) \wedge (p \supset r) \\ B_0 & = & (p \supset q) \supset p & \\ B_1 & = & p \supset r & \end{array}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{p,\, p^+, q^+, r^+, A \vdash p \rightarrow_0 p}\,{\scriptstyle \mathrm{Ax}}}{p,\, p^+, q^+, r^+, A \Rightarrow p}\,{\scriptstyle \mathrm{LC_0}}}{p,\, p^+, r^+, A \Rightarrow q}\,{\scriptstyle \mathrm{RC}}}{p^+, r^+, A \Rightarrow p \supset q}\,{\scriptstyle \mathrm{R_\supset}} \quad \dfrac{}{p^+, r^+, A \vdash p \rightarrow_0 p}\,{\scriptstyle \mathrm{Ax}}}{p^+, r^+, A, B_0 \rightarrow_0 p}\,{\scriptstyle \mathrm{L_\supset}}}{p^+, r^+, A; A \rightarrow_0 p}\,{\scriptstyle \mathrm{LC_0}}}{p^+, r^+, A \Rightarrow p}\,{\scriptstyle \mathrm{RC}} \quad \dfrac{}{r^+, A \vdash r \rightarrow_0 r}\,{\scriptstyle \mathrm{Ax}}}{r^+, A; B \rightarrow_0 r}\,{\scriptstyle \mathrm{L_\supset}}}{r^+, A; A \rightarrow_0 r}\,{\scriptstyle \mathrm{LC_0}}}{r^+, A \Rightarrow r}\,{\scriptstyle \mathrm{RC}}}{A \Rightarrow r}\,{\scriptstyle \mathrm{R_\supset}}}{\cdot \Rightarrow A \supset r}\,{\scriptstyle \mathrm{R_\supset}}$$

Translation in NC

$$
\begin{array}{rcl}
A & = & B_0 \wedge B_1 \quad = \quad ((p \supset q) \supset p) \wedge (p \supset r) \\
B_0 & = & (p \supset q) \supset p \\
B_1 & = & p \supset r
\end{array}
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{}{p^1, r^1, A \vdash A_0^1}\,{\scriptstyle u}}{p^1, r^1, A \vdash B_0^1}\,{\scriptstyle \to_E} \qquad \dfrac{\dfrac{\dfrac{}{p, p^1, q^1, r^1, A \vdash p^1}\,{\scriptstyle u}}{p, p^1, q^1, r^1, A \vdash q^1}\,{\scriptstyle \supset_E}}{p, p^1, r^1, A \vdash p \supset q^1}\,{\scriptstyle \supset_I}}{p^1, r^1, A \vdash p^1}\,{\scriptstyle \supset_E}}{\dfrac{\dfrac{}{r^1, A \vdash A_1^1}\,{\scriptstyle u}}{r^1, A \vdash B_1^1}\,{\scriptstyle \to_E}}
$$

$$
\dfrac{\dfrac{\dfrac{}{r^1, A \vdash r^1}\,{\scriptstyle u}}{r^1, A \vdash r^1}\,{\scriptstyle \supset_E}}{\dfrac{A \vdash r^1}{\vdash A \supset r^1}\,{\scriptstyle \supset_I}}\,{\scriptstyle \supset_E}
$$

Derivations built by `mjc`

Warning
Multisets are represented as sets (formulas are not duplicated) and dots denote empty multisets; $\neg A$ abbreviates $A \supset \bot$.

Formula label: gabbay0111

$$((c \supset a) \wedge ((c \supset a) \supset c)) \supset a$$

Derivation in MJcr

$$\begin{array}{lcll} A_0 & = & c \supset a & \\ A_1 & = & A_0 \supset c & = \quad (c \supset a) \supset c \\ A & = & A_0 \wedge A_1 & = \quad (c \supset a) \wedge ((c \supset a) \supset c) \end{array}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\supset c, c \to_0 c, c}\,\text{Ax}}{\supset c, c \to_0 c \mid a}\,\text{Build}}{c \Rightarrow a \mid a, c}\,R\supset}{\to \Rightarrow A_0 \mid a, c}\,L\supset \qquad \dfrac{\dfrac{}{\supset c, c \to_0 c \mid a}\,\text{Ax}}{\supset c \to_0 c \mid a}\,L\supset$$

$$\dfrac{\dfrac{\supset A_1 \to_0 c, a}{A_1 \Rightarrow c \mid a}\,L\!\to_0 \qquad \dfrac{}{\neg A_1 \mid a \to_0 a, \cdots}\,\text{Ax}}{\dfrac{\supset A_1 \mid A_0 \to_0 a, \cdots}{\supset \wedge A \to_0 a, \cdots}\,L\wedge_0}\,L\supset$$

$$\dfrac{\supset \wedge A \to_0 a, \cdots}{A \Rightarrow a, \cdots}\,L\!\to_0$$

$$\dfrac{A \Rightarrow a, \cdots}{\to \Rightarrow A \supset a, \cdots}\,R\supset$$

Translation in MJc

$$
\begin{array}{lcll}
A_0 & = & c \supset a & \\
A_1 & = & A_0 \supset c & = \quad (c \supset a) \supset c \\
A & = & A_0 \wedge A_1 & = \quad (c \supset a) \wedge ((c \supset a) \supset c)
\end{array}
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{c, a^+, c^+, A_1 ; c \neg_0 c^-}\,\mathrm{Ax}}{c, a^+, c^+, A_1 \Rightarrow c}\,\mathrm{LC_0}}{c, a^+, c^-, A_1 \Rightarrow a}\,\mathrm{RC}}{a^+, c^+, A_1 \Rightarrow A_0}\,\mathrm{R_\supset} \qquad \dfrac{}{a^+, c^+, A_1 ; c \neg_0 c}\,\mathrm{Ax}}{\dfrac{a^+, c^+, A_1, A_1 \neg_0 c}{a^+, c^+, A_1 \Rightarrow c}\,\mathrm{LC_0}}\,\mathrm{RC}}{\dfrac{a^+, A_1 A_0 \vdash}{a^+, A_1 \Rightarrow c}\,\mathrm{L_\wedge}} \qquad \dfrac{}{a^+, A ; a \neg_0 a}\,\mathrm{Ax}}{\dfrac{a^+, A \Rightarrow c}{\dfrac{a^+, A_1 A_0 \neg_0 a}{a^+, A_1 A \neg_0 a}\,\mathrm{L_\wedge}}\,\mathrm{L_\supset}}
$$

$$
\dfrac{\dfrac{a^+, A_1 A \neg_0 a}{a^+, A \Rightarrow a}\,\mathrm{LC_0}}{\dfrac{A \Rightarrow a}{\vdash A \supset a}\,\mathrm{R_\supset}}\,\mathrm{RC}
$$

Elimination of substitutions

$$\begin{array}{rcll} A_0 & = & c \supset a & \\ A_1 & = & A_0 \supset c & = \quad (c \supset a) \supset c \\ A & = & A_0 \wedge A_1 & = \quad (c \supset a) \wedge ((c \supset a) \supset c) \end{array}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{c, a^+, c^+, A|c \to_0 c}\,Ax}{c, a^+, c^+, A \Rightarrow c}\,\mathrm{L}\forall_0}{c, a^+, c^+, A \Rightarrow a}\,\mathrm{R}\subset}{a^+, c^+, A \Rightarrow A_0}\,\mathrm{R}\wedge \qquad \dfrac{}{a^+, c^+, A|c \to_0 c}\,Ax}{\dfrac{a^+, c^+, A_0|A_1 \to_0 c}{a^+, c^+, A|A \to_0 c}\,\mathrm{L}\forall_0}\,\mathrm{L}\wedge_1$$
$$\dfrac{a^+, c^+, A \Rightarrow c}{a^+, A \Rightarrow c}\,\mathrm{R}\subset \qquad \dfrac{}{a^+, A|a \to_0 a}\,Ax$$
$$\dfrac{a^+, A|A_0 \to_0 a}{a^+, A|A \to_0 a}\,\mathrm{L}\forall_0$$
$$\dfrac{a^+, A \Rightarrow a}{A \Rightarrow a}\,\mathrm{R}\subset$$
$$\vdash a \wedge A \supset a$$

Translation in NC

$$
\begin{array}{rcll}
A_0 & = & c \supset a & \\
A_1 & = & A_0 \supset c & = \quad (c \supset a) \supset c \\
A & = & A_0 \wedge A_1 & = \quad (c \supset a) \wedge ((c \supset a) \supset c)
\end{array}
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{}{a^{\uparrow}, A \vdash A_0^{\downarrow}}\,{\scriptstyle id}}{a^{\uparrow}, A \vdash A_0^{\downarrow}}\,{\scriptstyle \wedge_R}}{a^{\uparrow}, c^{\uparrow}, A \vdash A_0^{\downarrow}}\,{\scriptstyle id} \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{c, a^{\uparrow}, c^{\uparrow}, A \vdash c^{\downarrow}}\,{\scriptstyle id}}{c, a^{\uparrow}, c^{\uparrow}, A \vdash c^{\uparrow}}\,{\scriptstyle \supset_R}}{c, a^{\uparrow}, c^{\uparrow}, A \vdash a^{\uparrow}}\,{\scriptstyle \supset_E}}{a^{\uparrow}, c^{\uparrow}, A \vdash A_0^{\uparrow}}\,{\scriptstyle \supset_R}}{a^{\uparrow}, c^{\uparrow}, A \vdash c^{\downarrow}}\,{\scriptstyle \supset_E}}{a^{\uparrow}, c^{\uparrow}, A \vdash c^{\uparrow}}\,{\scriptstyle \supset_E}}{a^{\uparrow}, A \vdash c^{\uparrow}}\,{\scriptstyle \supset_E}}{\dfrac{\dfrac{a^{\uparrow}, A \vdash a^{\downarrow}}{a^{\uparrow}, A \vdash a^{\uparrow}}\,{\scriptstyle \supset_E}}{A \vdash a^{\uparrow}}\,{\scriptstyle \supset_E}}{\vdash A \supset a^{\uparrow}}\,{\scriptstyle \supset_I}
$$

Derivations built by `mjc`

Warning
Multisets are represented as sets (formulas are not duplicated) and dots denote empty multisets; $\neg A$ abbreviates $A \supset \bot$.

Formula label: gabbay0112

$$((((b_3 \supset a) \supset c) \wedge ((b_2 \supset a) \supset c)) \wedge (b_1 \supset (b_2 \supset c))) \wedge (c \supset a)) \supset a$$

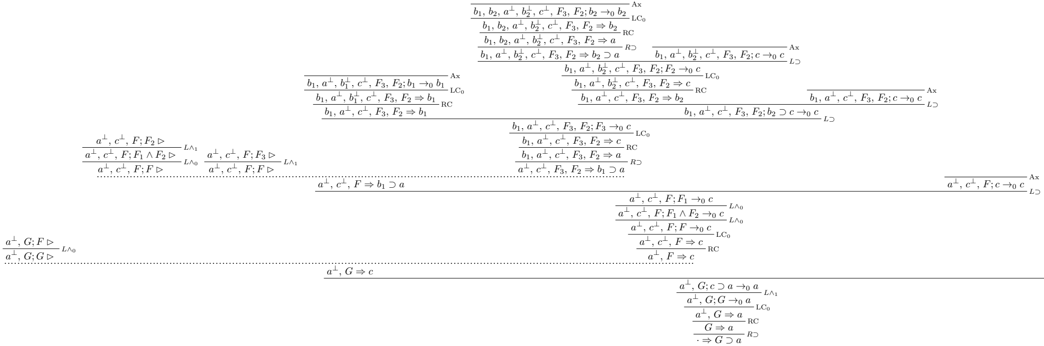
Derivation in MJcr

$$
\begin{array}{lcl}
F_1 & = & (b_3 \supset a) \supset c \\
F_2 & = & (b_2 \supset a) \supset c \\
F_3 & = & b_1 \supset (b_2 \supset c) \\
\mathcal{F} & = & (F_1 \wedge F_2) \wedge F_3 \quad = \quad (((b_3 \supset a) \supset c) \wedge ((b_2 \supset a) \supset c)) \wedge (b_1 \supset (b_2 \supset c)) \\
G & = & F \wedge (c \supset a) \quad = \quad ((((b_3 \supset a) \supset c) \wedge ((b_2 \supset a) \supset c)) \wedge (b_1 \supset (b_2 \supset c))) \wedge (c \supset a)
\end{array}
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{F_G, b_1 \to_0 b_2, a, c}\,{\scriptstyle Ax}}{b_1, F_2 \Rightarrow b_2, a, c}\,{\scriptstyle L\neg_0}}{b_1, F_2, F_G \Rightarrow a, c}\,{\scriptstyle RefLf}}{b_1, F_2 \Rightarrow b_2 \supset a, c}\,{\scriptstyle R\supset}}{b_1, F_2, F_G \Rightarrow c, a}\,{\scriptstyle L\neg_0}}{b_1, F_2 \Rightarrow b_2, a, c}\,{\scriptstyle RefLf}}{b_1, F_2, F_G \Rightarrow a, c}\,{\scriptstyle R\supset}}{F_2, F_G \Rightarrow b_1 \supset a, a, c}\,{\scriptstyle R\supset}}{F_1, F_2, F_G \Rightarrow a, a, c}\,{\scriptstyle L\wedge_0}}{F, F_G \Rightarrow a, c}\,{\scriptstyle L\wedge_0}}{F \Rightarrow c, a}\,{\scriptstyle L\neg_0}}{\Rightarrow F, c, a}\,{\scriptstyle R\supset}}{\Rightarrow G \supset a, c}\,{\scriptstyle R\supset}
\qquad
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{b_1, F_G, c \to_0 c, a}\,{\scriptstyle Ax}}{b_1, F_2, F_G \Rightarrow c, a}\,{\scriptstyle L\supset}}{b_1, F_2, F_G \Rightarrow b_2, a, c}\,{\scriptstyle RefLf}}{b_1, F_2, F_G \Rightarrow b_2 \supset a, c}\,{\scriptstyle R\supset}}{F_2, F_G \Rightarrow b_1 \supset a, a, c}\,{\scriptstyle R\supset}}{F_1, F_2, F_G \Rightarrow a, a, c}\,{\scriptstyle L\wedge_0}}{F, F_G \Rightarrow a, c}\,{\scriptstyle L\wedge_0}}{F \Rightarrow c, a}\,{\scriptstyle L\neg_0}}{\Rightarrow F, c, a}\,{\scriptstyle R\supset}}{\Rightarrow G \supset a, c}\,{\scriptstyle R\supset}
\qquad
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{b_1, F_G, c \to_0 c, a}\,{\scriptstyle Ax}}{b_1, F_2, F_G \Rightarrow c, a}\,{\scriptstyle L\supset}}{b_1, F_2, F_G \Rightarrow b_2, a, c}\,{\scriptstyle RefLf}}{b_1, F_2 \Rightarrow b_2 \supset a, c}\,{\scriptstyle R\supset}}{F_2, F_G \Rightarrow b_1 \supset a, a, c}\,{\scriptstyle R\supset}}{F_1, F_2, F_G \Rightarrow a, a, c}\,{\scriptstyle L\wedge_0}}{F, F_G \Rightarrow a, c}\,{\scriptstyle L\wedge_0}}{F \Rightarrow c, a}\,{\scriptstyle L\neg_0}}{\Rightarrow F, c, a}\,{\scriptstyle R\supset}}{\Rightarrow G \supset a, c}\,{\scriptstyle R\supset}
\qquad
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{b_1, F_G, c \to_0 c, a}\,{\scriptstyle Ax}}{b_1, F_2, F_G \Rightarrow c, a}\,{\scriptstyle L\supset}}{b_1, F_2 \Rightarrow b_2, a, c}\,{\scriptstyle RefLf}}{b_1, F_2 \Rightarrow b_2 \supset a, c}\,{\scriptstyle R\supset}}{F_2, F_G \Rightarrow b_1 \supset a, a, c}\,{\scriptstyle R\supset}}{F_1, F_2, F_G \Rightarrow a, a, c}\,{\scriptstyle L\wedge_0}}{F, F_G \Rightarrow a, c}\,{\scriptstyle L\wedge_0}}{F \Rightarrow c, a}\,{\scriptstyle L\neg_0}}{\Rightarrow F, c, a}\,{\scriptstyle R\supset}}{\Rightarrow G \supset a, c}\,{\scriptstyle R\supset}
$$

Translation in MJc

$$\begin{array}{lclcl}
F_1 & = & (b_1 \supset a) \supset c & & \\
F_2 & = & (b_2 \supset a) \supset c & & \\
F_3 & = & b_1 \supset (b_2 \supset c) & & \\
F & = & (F_1 \wedge F_2) \wedge F_3 & = & (((b_1 \supset a) \supset c) \wedge ((b_2 \supset a) \supset c)) \wedge (b_1 \supset (b_2 \supset c)) \\
G & = & F \wedge (c \supset a) & = & ((((b_1 \supset a) \supset c) \wedge ((b_2 \supset a) \supset c)) \wedge (b_1 \supset (b_2 \supset c))) \wedge (c \supset a)
\end{array}$$


Ax
$b_1, b_2, a^\perp, b_2^\perp, c^\perp, F_3, F_2; b_2 \to_0 b_2$
LC$_0$
$b_1, b_2, a^\perp, b_2^\perp, c^\perp, F_3, F_2 \Rightarrow b_2$
RC
$b_1, b_2, a^\perp, b_2^\perp, c^\perp, F_3, F_2 \Rightarrow a$
$R\supset$
$b_1, a^\perp, b_2^\perp, c^\perp, F_3, F_2 \Rightarrow b_2 \supset a$
Ax
$b_1, a^\perp, b_2^\perp, c^\perp, F_3, F_2; c \to_0 c$
$L\supset$
$b_1, a^\perp, b_2^\perp, c^\perp, F_3, F_2; F_2 \to_0 c$
LC$_0$
$b_1, a^\perp, b_2^\perp, c^\perp, F_3, F_2 \Rightarrow c$
RC
$b_1, a^\perp, c^\perp, F_3, F_2 \Rightarrow b_2$
Ax
$b_1, a^\perp, b_1^\perp, c^\perp, F_3, F_2; b_1 \to_0 b_1$
LC$_0$
$b_1, a^\perp, b_1^\perp, c^\perp, F_3, F_2 \Rightarrow b_1$
RC
$b_1, a^\perp, c^\perp, F_3, F_2 \Rightarrow b_1$
Ax
$b_1, a^\perp, c^\perp, F_3, F_2; c \to_0 c$
$L\supset$
$b_1, a^\perp, c^\perp, F_3, F_2; b_2 \supset c \to_0 c$
$L\supset$
$b_1, a^\perp, c^\perp, F_3, F_2; F_3 \to_0 c$
LC$_0$
$b_1, a^\perp, c^\perp, F_3, F_2 \Rightarrow c$
RC
$b_1, a^\perp, c^\perp, F_3, F_2 \Rightarrow a$
$R\supset$
$a^\perp, c^\perp, F_3, F_2 \Rightarrow b_1 \supset a$
$a^\perp, c^\perp, F; F_2 \triangleright$
$L\wedge_1$
$a^\perp, c^\perp, F; F_1 \wedge F_2 \triangleright$
$L\wedge_0$
$a^\perp, c^\perp, F; F \triangleright$
$a^\perp, c^\perp, F; F_3 \triangleright$
$L\wedge_1$
$a^\perp, c^\perp, F; F \triangleright$
$a^\perp, c^\perp, F \Rightarrow b_1 \supset a$
Ax
$a^\perp, c^\perp, F; c \to_0 c$
$L\supset$
$a^\perp, c^\perp, F; F_1 \to_0 c$
$L\wedge_0$
$a^\perp, c^\perp, F; F_1 \wedge F_2 \to_0 c$
$L\wedge_0$
$a^\perp, c^\perp, F; F \to_0 c$
LC$_0$
$a^\perp, c^\perp, F \Rightarrow c$
RC
$a^\perp, F \Rightarrow c$
$a^\perp, G; F \triangleright$
$L\wedge_0$
$a^\perp, G; G \triangleright$
$a^\perp, G \Rightarrow c$
$a^\perp, G; c \supset a \to_0 a$
$L\wedge_1$
$a^\perp, G; G \to_0 a$
LC$_0$
$a^\perp, G \Rightarrow a$
RC
$G \Rightarrow a$
$R\supset$
$\cdot \Rightarrow G \supset a$

$$\dfrac{\dfrac{}{a^{\perp},\, G; a \rightarrow_0 a}\,\text{Ax}}{}\,L\supset$$

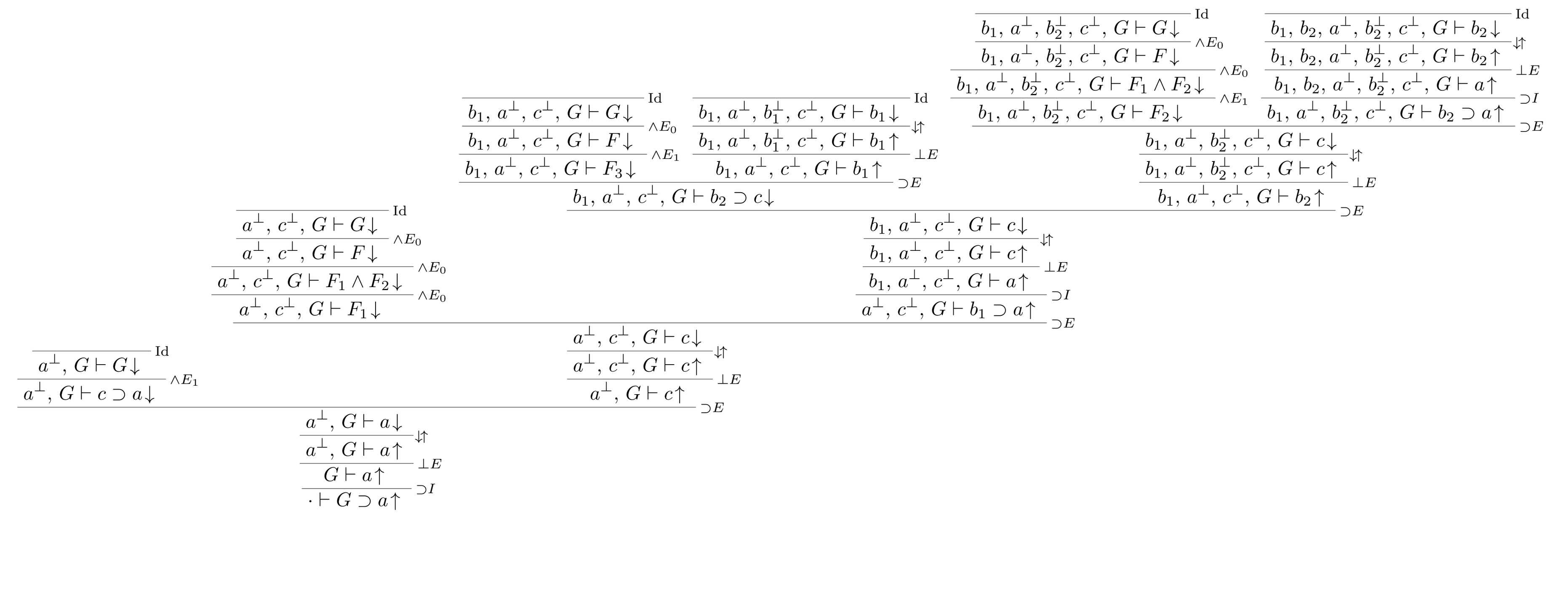
Elimination of substitutions

$$
\begin{array}{lclcl}
F_1 & = & (b_1 \supset a) \supset c & & \\
F_2 & = & (b_2 \supset a) \supset c & & \\
F_3 & = & b_1 \supset (b_2 \supset c) & & \\
F & = & (F_1 \wedge F_2) \wedge F_3 & = & (((b_1 \supset a) \supset c) \wedge ((b_2 \supset a) \supset c)) \wedge (b_1 \supset (b_2 \supset c)) \\
G & = & F \wedge (c \supset a) & = & ((((b_1 \supset a) \supset c) \wedge ((b_2 \supset a) \supset c)) \wedge (b_1 \supset (b_2 \supset c))) \wedge (c \supset a)
\end{array}
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{b_1, a^\perp, b_1^\perp, c^\perp, G; b_1 \to_0 b_1}\,\mathrm{Ax}}{b_1, a^\perp, b_1^\perp, c^\perp, G \Rightarrow b_1}\,\mathrm{LC}_0}{b_1, a^\perp, c^\perp, G \Rightarrow b_1}\,\mathrm{RC} \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{b_1, b_2, a^\perp, b_2^\perp, c^\perp, G; b_2 \to_0 b_2}\,\mathrm{Ax}}{b_1, b_2, a^\perp, b_2^\perp, c^\perp, G \Rightarrow b_2}\,\mathrm{LC}_0}{b_1, b_2, a^\perp, b_2^\perp, c^\perp, G \Rightarrow a}\,\mathrm{RC}}{b_1, a^\perp, b_2^\perp, c^\perp, G \Rightarrow b_2 \supset a}\,R{\supset} \quad \dfrac{}{b_1, a^\perp, b_2^\perp, c^\perp, G; c \to_0 c}\,\mathrm{Ax}}{b_1, a^\perp, b_2^\perp, c^\perp, G; F_2 \to_0 c}\,L{\supset}}{b_1, a^\perp, b_2^\perp, c^\perp, G; F_1 \wedge F_2 \to_0 c}\,L{\wedge_1}}{b_1, a^\perp, b_2^\perp, c^\perp, G; F \to_0 c}\,L{\wedge_0}}{b_1, a^\perp, b_2^\perp, c^\perp, G; G \to_0 c}\,L{\wedge_0}}{b_1, a^\perp, b_2^\perp, c^\perp, G \Rightarrow c}\,\mathrm{LC}_0}{b_1, a^\perp, c^\perp, G \Rightarrow b_2}\,\mathrm{RC} \quad \dfrac{}{b_1, a^\perp, c^\perp, G; c \to_0 c}\,\mathrm{Ax}}{b_1, a^\perp, c^\perp, G; b_2 \supset c \to_0 c}\,L{\supset}}{b_1, a^\perp, c^\perp, G; F_3 \to_0 c}\,L{\supset}}{b_1, a^\perp, c^\perp, G; F \to_0 c}\,L{\wedge_1}}{b_1, a^\perp, c^\perp, G; G \to_0 c}\,L{\wedge_0}}{b_1, a^\perp, c^\perp, G \Rightarrow c}\,\mathrm{LC}_0}{b_1, a^\perp, c^\perp, G \Rightarrow a}\,\mathrm{RC}}{a^\perp, c^\perp, G \Rightarrow b_1 \supset a}\,R{\supset} \quad \dfrac{}{a^\perp, c^\perp, G; c \to_0 c}\,\mathrm{Ax}}{a^\perp, c^\perp, G; F_1 \to_0 c}\,L{\supset}}{a^\perp, c^\perp, G; F_1 \wedge F_2 \to_0 c}\,L{\wedge_0}}{a^\perp, c^\perp, G; F \to_0 c}\,L{\wedge_0}}{a^\perp, c^\perp, G; G \to_0 c}\,L{\wedge_0}}{a^\perp, c^\perp, G \Rightarrow c}\,\mathrm{LC}_0}{a^\perp, G \Rightarrow c}\,\mathrm{RC} \quad \dfrac{}{a^\perp, G; a \to_0 a}\,\mathrm{Ax}}{a^\perp, G; c \supset a \to_0 a}\,L{\supset}}{a^\perp, G; G \to_0 a}\,L{\wedge_1}}{a^\perp, G \Rightarrow a}\,\mathrm{LC}_0}{G \Rightarrow a}\,\mathrm{RC}}{\cdot \Rightarrow G \supset a}\,R{\supset}
$$

Translation in NC

$$\begin{array}{lclcl} F_1 & = & (b_1 \supset a) \supset c & & \\ F_2 & = & (b_2 \supset a) \supset c & & \\ F_3 & = & b_1 \supset (b_2 \supset c) & & \\ F & = & (F_1 \wedge F_2) \wedge F_3 & = & (((b_1 \supset a) \supset c) \wedge ((b_2 \supset a) \supset c)) \wedge (b_1 \supset (b_2 \supset c)) \\ G & = & F \wedge (c \supset a) & = & ((((b_1 \supset a) \supset c) \wedge ((b_2 \supset a) \supset c)) \wedge (b_1 \supset (b_2 \supset c))) \wedge (c \supset a) \end{array}$$



Derivations built by `mjc`

Warning
Multisets are represented as sets (formulas are not duplicated) and dots denote empty multisets; $\neg A$ abbreviates $A \supset \bot$.

Formula label: gabbay0113

$$(((p \supset q) \supset p) \wedge (p \supset r)) \supset r$$

Derivation in MJcr

$$\begin{array}{lcll} A & = & A_0 \wedge A_1 & = \quad (((p \supset q) \supset p) \wedge (p \supset r) \\ A_0 & = & (p \supset q) \supset p & \\ A_1 & = & p \supset r & \end{array}$$

$$\dfrac{\dfrac{\dfrac{}{\neg\neg p \to p, p, q \Rightarrow}\,{\scriptstyle hyhI}}{\dfrac{p \Rightarrow q, p, r}{\neg\neg p \supset q, p, r}\,{\scriptstyle R\supset}}\,{\scriptstyle \wedge A} \qquad \dfrac{}{\neg\neg p \to p, r \Rightarrow}\,{\scriptstyle \wedge A}}{\dfrac{\dfrac{\neg\neg A_0 \to p, r \Rightarrow}{A_0 \Rightarrow p, r}\,{\scriptstyle LL_0} \qquad \dfrac{}{\neg A_0, r \to_0 r, \cdot}\,{\scriptstyle \wedge A}}{\dfrac{\neg A_0, A_0 \to_0 r, \cdot}{\neg\neg A \to_0 r, \cdot}\,{\scriptstyle L\wedge_0}}\,{\scriptstyle L\supset}}$$
$$\dfrac{\neg\neg A \Rightarrow r, \cdot}{\neg\neg A \Rightarrow r, \cdot}\,{\scriptstyle LL_0}$$
$$\neg\neg A \Rightarrow r, \cdot\,{\scriptstyle R\supset}$$

Translation in MJc

$$
\begin{array}{lcll}
A & = & A_0 \wedge A_1 & = \quad ((p \supset q) \supset p) \wedge (p \supset r) \\
A_0 & = & (p \supset q) \supset p & \\
A_1 & = & p \supset r &
\end{array}
$$

$$
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\dfrac{}{p, p^{\sharp}, q^{\sharp}, r^{\sharp}, A_0 \Rightarrow p}\,{\scriptstyle Ax}
}{p, p^{\sharp}, r^{\sharp}, A_0 \Rightarrow q}\,{\scriptstyle LC_{\supset}}
}{p^{\sharp}, r^{\sharp}, A_0 \Rightarrow p \supset q}\,{\scriptstyle RC}
\qquad
\dfrac{}{p^{\sharp}, r^{\sharp}, A_0, p \Rightarrow p}\,{\scriptstyle Ax}
}{p^{\sharp}, r^{\sharp}, A_0, A_0 \Rightarrow p}\,{\scriptstyle LC_{\supset}}
}{r^{\sharp}, A, A_0 \Rightarrow p}\,{\scriptstyle LC_{\wedge_0}}
\qquad
\dfrac{}{r^{\sharp}, A, r \Rightarrow r}\,{\scriptstyle Ax}
}{r^{\sharp}, A, A \Rightarrow r}\,{\scriptstyle LC_{\wedge_1}}
}{r^{\sharp}, A, A \Rightarrow r}\,{\scriptstyle LC_{\wedge}}
}{A \Rightarrow r}\,{\scriptstyle RC}
}{\Rightarrow A \supset r}\,{\scriptstyle RC_{\supset}}
$$

Elimination of substitutions

$$\begin{array}{lcll} A & = & A_0 \wedge A_1 & = \quad ((p \supset q) \supset p) \wedge (p \supset r) \\ A_0 & = & (p \supset q) \supset p & \\ A_1 & = & p \supset r & \end{array}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{p,\, p^+, q^+, r^+, A \vdash p \rightsquigarrow p}\,\mathrm{Ax}}{p,\, p^+, q^+, r^+, A \Rightarrow p}\,\mathrm{LC_+}}{p,\, p^+, r^+, A \Rightarrow q}\,\mathrm{RC}}{p^+, r^+, A \Rightarrow p \supset q}\,R_\supset \qquad \dfrac{}{p^+, r^+, A|p \rightsquigarrow p}\,\mathrm{Ax}}{\dfrac{\dfrac{\dfrac{\dfrac{p^+, r^+, A, A_0 \rightsquigarrow p}{p^+, r^+, A|A \rightsquigarrow p}\,L\mathrm{A_+}}{p^+, r^+, A \Rightarrow p}\,\mathrm{RC} \qquad \dfrac{}{r^+, A|r \rightsquigarrow r}\,\mathrm{Ax}}{r^+, A \Rightarrow p}\,L_\supset}{\dfrac{\dfrac{r^+, A, A_1 \rightsquigarrow r}{r^+, A|A \rightsquigarrow r}\,L\mathrm{A_+}}{r^+, A \Rightarrow r}\,\mathrm{LC_+}}{A \Rightarrow r}\,\mathrm{RC}}{\cdot \Rightarrow A \supset r}\,R_\supset$$

Translation in NC

$$
\begin{array}{rcl}
A & = & A_0 \wedge A_1 \quad = \quad ((p \supset q) \supset p) \wedge (p \supset r) \\
A_0 & = & (p \supset q) \supset p \\
A_1 & = & p \supset r
\end{array}
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{p^1, r^1, A \vdash A_1^1}\,\text{ii}}{p^1, r^1, A \vdash A_1^1}\,\text{iii} \qquad \dfrac{\dfrac{\dfrac{}{p, p^1, q^1, r^1, A \vdash p^1}\,\text{ii}}{p, p^1, q^1, r^1, A \vdash p^1}\,\text{iv}}{p, p^1, r^1, A \vdash p^1}\,\text{iv}}{p^1, r^1, A \vdash p \supset q^1}\,\text{v}}{p^1, r^1, A \vdash p^1}\,\text{vi}}{p^1, r^1, A \vdash p^1}\,\text{vii}}{r^1, A \vdash A_1^1}\,\text{ii}}{r^1, A \vdash A_1^1}\,\text{vii}
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{}{r^1, A \vdash A_1^1}\,\text{ii}}{r^1, A \vdash A_1^1}\,\text{iii} \qquad \dfrac{\dfrac{\dfrac{}{p^1, r^1, A \vdash p^1}\,\text{ii}}{p^1, r^1, A \vdash p^1}\,\text{iv}}{r^1, A \vdash p^1}\,\text{vi}}{r^1, A \vdash r^1}\,\text{vii}}{\dfrac{r^1, A \vdash r^1}{\vdash A \supset r^1}\,\text{vi}}\,\text{vii}
$$

Derivations built by `mjc`

Warning
Multisets are represented as sets (formulas are not duplicated) and dots denote empty multisets; $\neg A$ abbreviates $A \supset \bot$.

Formula label: gabbay0114

$$((a \supset b) \supset b) \supset (\neg a \supset b)$$

Derivation in MJcr

$$\begin{array}{rcl} F & = & (a \supset b) \supset b \\ G & = & \neg a \supset b \end{array}$$

$$\dfrac{\dfrac{\dfrac{}{\cdot ; \Box a \to a_0 \Rightarrow a_0}\,Ax}{\cdot \Rightarrow a \supset b, a_0}\,RelI}{a \Rightarrow b, a_0, b_\sqsubset \Rightarrow a_\sqsubset}\,Ax_\sqsubset \qquad \dfrac{}{\cdot ; b \to_0 b \Rightarrow a}\,Ax_\sqsubset}{\cdot \Rightarrow a \supset b, a, b}\,RelI \qquad \dfrac{}{F ; \bot \to_1 b_\sqsubset \Rightarrow a_\sqsubset}\,L\bot_\sqsubset}{\cdot ; F \to_0 b \Rightarrow a}\,RelI \qquad \dfrac{}{F ; \Box \bot \to_1 b_\sqsubset \Rightarrow a_\sqsubset}\,L\bot_\sqsubset}{F \Rightarrow a, b}\,\to_1 \qquad \dfrac{}{F ; \Box \bot \to_1 b_\sqsubset \Rightarrow a_\sqsubset}\,L\bot_\sqsubset$$

$$\dfrac{F ; \cdot \to \neg a \to_1 b_\sqsubset \Rightarrow a_\sqsubset}{\neg a, F \Rightarrow b, a_\sqsubset}\,\to_1$$

$$\dfrac{\neg a, F \Rightarrow b, a_\sqsubset}{F \Rightarrow G, a_\sqsubset}\,\to_1$$

$$\dfrac{F \Rightarrow G, a_\sqsubset}{\cdot \Rightarrow F \supset G, a_\sqsubset}\,\to_\supset$$

Translation in MJc

$$\begin{array}{rcl} F & = & (a \supset b) \supset b \\ G & = & \neg a \supset b \end{array}$$

$$\dfrac{\dfrac{\dfrac{}{a, a^+, b^+, \neg a, F \Rightarrow \neg a, a}\,Ax}{a, a^+, b^+, \neg a, F \Rightarrow a}\,LC_0}{\dfrac{a, a^+, b^+, \neg a, F \Rightarrow a}{a^+, b^+, \neg a, F \Rightarrow a \supset b}\,RC}\,RC^+ \qquad \dfrac{}{a^+, b^+, \neg a, F, b \Rightarrow b}\,Ax}{\dfrac{a^+, b^+, \neg a, F, F \Rightarrow b}{a^+, b^+, \neg a, F \Rightarrow b}\,LC_0}\,LC^+$$
$$\dfrac{a^+, b^+, \neg a, F \Rightarrow b}{b^+, \neg a, F \Rightarrow a}\,RC \qquad \dfrac{}{b^+, \neg a, F, \bot \Rightarrow_I b}\,L\bot$$
$$\dfrac{b^+, \neg a, F \Rightarrow \neg_I b}{\neg a, F \Rightarrow b}\,LC_1$$
$$\dfrac{\neg a, F \Rightarrow b}{F \Rightarrow G}\,RC_1$$
$$\dfrac{}{\cdot \Rightarrow F \supset G}\,RC_2$$

Translation in NC

$$
\begin{array}{rcl}
F & = & (a \supset b) \supset b \\
G & = & \neg a \supset b
\end{array}
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{}{a, a^+, b^0, \neg a, F \vdash a}\,\mathrm{Id}}{a, a^+, b^0, \neg a, F \vdash a^0_\perp}\,{-}Z}{a, a^+, b^0, \neg a, F \vdash a^0}\,{-}E}{a, a^+, b^+, \neg a, F \vdash b^+}\,{-}I^0}{a^+, b^+, \neg a, F \vdash F_\perp}\,\mathrm{Id} \quad \dfrac{\dfrac{\dfrac{\dfrac{}{a^+, b^0, \neg a, F \vdash b}\,\mathrm{Id}}{a^+, b^0, \neg a, F \vdash b^0_\perp}\,{-}Z}{a^+, b^0, \neg a, F \vdash b^0}\,{-}E}{a^+, b^+, \neg a, F \vdash b^+}\,{-}I^0}{a^+, b^+, \neg a, F \vdash a \supset b^+}\,{\supset}I}
$$

$$
\dfrac{\dfrac{}{b^0, \neg a, F \vdash \neg a}\,\mathrm{Id}}{\dfrac{\dfrac{b^0, \neg a, F \vdash \bot}{\neg a, F \vdash b^0}\,{\perp}E}{F \vdash G^+}\,{-}I}{\vdash F \supset G^+}\,{\supset}I} \qquad \dfrac{}{b^0, \neg a, F \vdash a^0_\perp}\,{-}E
$$

Derivations built by `mjc`

Warning

Multisets are represented as sets (formulas are not duplicated) and dots denote empty multisets; $\neg A$ abbreviates $A \supset \bot$.

Formula label: `or1`

$$((p_0 \supset q) \wedge (p_1 \supset q)) \supset ((p_0 \vee p_1) \supset q)$$

Derivation in MJcr

$$\begin{array}{rcl}
A & = & A_0 \wedge A_1 \quad = \quad (p_0 \supset q) \wedge (p_1 \supset q) \\
A_0 & = & p_0 \supset q \\
A_1 & = & p_1 \supset q \\
B & = & C \supset q \quad = \quad (p_0 \vee p_1) \supset q \\
C & = & p_0 \vee p_1
\end{array}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\cdot \vdash p_1 \to p_0 \; p_1 \; p_0 \to q}\,{\scriptstyle \mathrm{Ax}}}{p_1 \Rightarrow p_1 \; p_0 \; q}\,{\scriptstyle \mathrm{Ax}}}{A_1 \vdash p_0 \to p_0 \; q}\,{\scriptstyle \mathrm{Ax}}}{p_0 \cdot A_1 \Rightarrow p_0 \; q}\,{\scriptstyle \mathrm{Ax}}}{A_1 \cdot C \to p_0 \; q}\,{\scriptstyle \mathrm{Ax}}}{A_1 \cdot C \Rightarrow p_0 \; q}\,{\scriptstyle \mathrm{Ax}}}{C \cdot A_1 \cdot A_0 \to p_0 \; q}\,{\scriptstyle \mathrm{Ax}}}{C \cdot A_1 \cdot A_0 \to q}\,{\scriptstyle \mathrm{Ax}}}{C \cdot A_1 \cdot A_0 \to q}\,{\scriptstyle \mathrm{Ax}}}{A \cdot C \Rightarrow q}\,{\scriptstyle \mathrm{Ax}}}{A \Rightarrow B}\,{\scriptstyle \mathrm{Ax}}}{\Rightarrow A \supset B}\,{\scriptstyle \mathrm{Ax}}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\cdot \vdash p_1 \to p_1 \; p_1 \; p_0 \to q}\,{\scriptstyle \mathrm{Ax}}}{p_1 \Rightarrow p_1 \; p_0 \; q}\,{\scriptstyle \mathrm{Ax}}}{p_1 \cdot A_1 \to q \; p_0}\,{\scriptstyle \mathrm{Ax}}}{A_1 \cdot C \to q}\,{\scriptstyle \mathrm{Ax}}}{C \cdot A_1 \cdot A_0 \to q}\,{\scriptstyle \mathrm{Ax}}}{C \cdot A_1 \cdot A_0 \to q}\,{\scriptstyle \mathrm{Ax}}}{A \cdot C \Rightarrow q}\,{\scriptstyle \mathrm{Ax}}}{A \Rightarrow B}\,{\scriptstyle \mathrm{Ax}}}{\Rightarrow A \supset B}\,{\scriptstyle \mathrm{Ax}}$$

Translation in MJc

$$
\begin{array}{lcll}
A & = & A_0 \wedge A_1 & (p_0 \supset q) \wedge (p_1 \supset q) \\
A_0 & = & p_0 \supset q & \\
A_1 & = & p_1 \supset q & \\
B & = & C \supset q & (p_0 \vee p_1) \supset q \\
C & = & p_0 \vee p_1 &
\end{array}
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{}{p_1, p_0^*, p_1^*, q^*, A_1, C, p_1 \to p_1}\,{\scriptstyle Ax}}{p_1, p_0^*, p_1^*, q^*, A_1, C \Rightarrow p_1}\,{\scriptstyle WC}}{p_1, p_0^*, q^*, A_1, C \Rightarrow p_1}\,{\scriptstyle LC_1} \qquad \dfrac{}{p_1, p_0^*, q^*, A_1, C, q \to q}\,{\scriptstyle Ax}}{p_1, p_0^*, q^*, A_1, C, A_1 \to q}\,{\scriptstyle L\supset}}{p_1, p_0^*, q^*, A_1, C \Rightarrow q}\,{\scriptstyle LC_1}}{p_1, p_0^*, q^*, A_1, C \Rightarrow p_1}\,{\scriptstyle WC} \qquad \dfrac{}{p_1, p_0^*, q^*, A_1, C \to p_1}\,{\scriptstyle Ax}}{p_1, p_0^*, q^*, A_1, C \Rightarrow p_1}\,{\scriptstyle LC_1}}{p_0^*, q^*, A_1, C, C \to p_0}\,{\scriptstyle LC_1}}{p_0^*, q^*, A_1, C \Rightarrow p_0}\,{\scriptstyle WC}}{q^*, A, C, A_1 \rhd \dots \rhd q^*, A, C, A \rhd}\,{\scriptstyle L\wedge_1} \qquad \dfrac{}{q^*, A, C \to p_0}\,{\scriptstyle Ax} \qquad \dfrac{}{q^*, A, C, q \to q}\,{\scriptstyle Ax}}{q^*, A, C, A_0 \to q}\,{\scriptstyle L\wedge_0}}{q^*, A, C, A \to q}\,{\scriptstyle LC_0}}{q^*, A, C \Rightarrow q}\,{\scriptstyle WC}}{A, C \Rightarrow q}\,{\scriptstyle R\supset}}{A \Rightarrow B}\,{\scriptstyle R\supset}}{\cdot \Rightarrow A \supset B}\,{\scriptstyle R\supset}
$$

Elimination of substitutions

$$
\begin{array}{lcll}
A & = & A_0 \wedge A_1 & (p_0 \supset q) \wedge (p_1 \supset q) \\
A_0 & = & p_0 \supset q & \\
A_1 & = & p_1 \supset q & \\
B & = & C \supset q & (p_0 \vee p_1) \supset q \\
C & = & p_0 \vee p_1 &
\end{array}
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{}{p_1, p_0^1, p_1^1, q^1, A, C, p_1 \to_0 p_1}\,{}_{Ax}}{p_1, p_0^1, p_1^1, q^1, A, C \Rightarrow p_1}\,{}_{LC_0} \quad \dfrac{}{p_1, p_0^1, q^1, A, C, q \to_0 q}\,{}_{Ax}}{p_1, p_0^1, q^1, A, C \Rightarrow p_1}\,{}_{RC} \quad \dfrac{}{p_1, p_0^1, q^1, A, C, q \to_0 q}\,{}_{Ax}}{\dfrac{p_1, p_0^1, q^1, A, C, A_1 \to_0 q}{p_1, p_0^1, q^1, A, C, A \to_0 q}\,{}_{LC_0}}\,{}_{LC_1}}{\dfrac{p_1, p_0^1, q^1, A, C \Rightarrow q}{p_1, p_0^1, q^1, A, C \Rightarrow p_0}\,{}_{LC_0}}\,{}_{RC}}{p_0, p_0^1, q^1, A, C \Rightarrow p_0}\,{}_{Ax}
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{}{p_0^1, q^1, A, C, C \to_0 p_0}\,{}_{Ax}}{p_0^1, q^1, A, C \Rightarrow p_0}\,{}_{LC_0}}{q^1, A, C \Rightarrow p_0}\,{}_{RC} \quad \dfrac{}{q^1, A, C, q \to_0 q}\,{}_{Ax}}{\dfrac{q^1, A, C, A_0 \to_0 q}{q^1, A, C, A \to_0 q}\,{}_{LC_0}}\,{}_{LC_1}}{\dfrac{q^1, A, C \Rightarrow q}{A, C \Rightarrow q}\,{}_{RC}}{\dfrac{}{A \Rightarrow B}\,{}_{RC}}{\cdot \Rightarrow A \supset B}\,{}_{RC}
$$

Translation in NC

$$
\begin{array}{lcll}
A & = & A_0 \wedge A_1 & (p_0 \supset q) \wedge (p_1 \supset q) \\
A_0 & = & p_0 \supset q & \\
A_1 & = & p_1 \supset q & \\
B & = & C \supset q & (p_0 \vee p_1) \supset q \\
C & = & p_0 \vee p_1 &
\end{array}
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{q^+, A, C \rhd A_1}\,\mathrm{id}}{q^+, A, C \rhd A_0}\,{\wedge}E_0 \quad \dfrac{\dfrac{}{p_0^-, q^+, A, C \rhd C_0^+}\,\mathrm{id}}{p_0 \supset p_0^-, q^+, A, C \rhd p_0^+}\,{\supset}E}{q^+, A, C \rhd q_0^+}\,{\supset}E}{q^+, A, C \rhd q_0^-}\,{\supset}I}{A \rhd B^+}\,{\supset}I}{\top \supset A \supset B^+}\,{\supset}I
\qquad
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{p_1 \supset p_1^-, q^+, A, C \rhd A_1^-}\,\mathrm{id}}{p_1 \supset p_1^-, q^+, A, C \rhd A_1}\,{\supset}E_1 \quad \dfrac{\dfrac{}{p_0 \supset p_0^-, q^+, A, C \rhd p_0}\,\mathrm{id}}{p_0 \supset p_0^-, q^+, A, C \rhd p_0^+}\,{\supset}E}{p_0 \supset p_0^-, q^+, A, C \rhd q_1}\,{\supset}I}{p_0 \supset p_0^-, q^+, A, C \rhd q_1^-}\,{\supset}I}{q^+, A, C \rhd q_1^+}\,{\supset}E
\qquad
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{p_1 \supset p_1^-, q^+, A, C \rhd p_1^-}\,\mathrm{id}}{p_1 \supset p_1^-, q^+, A, C \rhd p_1}\,{\supset}E_1 \quad \dfrac{\dfrac{}{p_0 \supset p_0^-, q^+, A, C \rhd p_0}\,\mathrm{id}}{p_0 \supset p_0^-, q^+, A, C \rhd p_0^+}\,{\supset}E}{p_0 \supset p_0^-, q^+, A, C \rhd q_1}\,{\supset}I}{p_0 \supset p_0^-, q^+, A, C \rhd q_1^-}\,{\supset}I}{q^+, A, C \rhd q_1^+}\,{\supset}E
$$

Derivations built by `mjc`

Warning

Multisets are represented as sets (formulas are not duplicated) and dots denote empty multisets; $\neg A$ abbreviates $A \supset \bot$.

Formula label: `or2`

$$((p \vee q) \supset r) \supset (p \vee (q \supset r))$$

Derivation in MJer

$$\begin{array}{rcl} A & = & (p \vee q) \supset r \\ B & = & p \vee (q \supset r) \end{array}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\cdot \subset \cdot q \to q \in p, r}\,\mathrm{Ax}}{q \Rightarrow p, p, q, r}\,\mathrm{Bot}}{q \Rightarrow p \vee q, p, r}\,\mathrm{Ax}}{q \subset A \to r \in p, r}\,\mathrm{Ax}}{q \subset A \to r \in p}\,\mathrm{Ax}}{A \Rightarrow q \supset r \in p}\,\mathrm{Bot}}{A \Rightarrow p \vee q \supset r}\,\mathrm{Bot}}{\cdot \to A \supset B \cdot}\,\mathrm{Ax}$$

Translation in MJc

$$\begin{array}{rcl} A & = & (p \vee q) \supset r \\ B & = & p \vee (q \supset r) \end{array}$$

$$\dfrac{\dfrac{\dfrac{}{q,\, p^+,\, q^+,\, r^+,\, (q \supset r)^+,\, (p \vee q)^+,\, B^+,\, A \Rightarrow_{\mathbf{C}} q}\,{}^{\mathrm{Ax}}}{q,\, p^+,\, q^+,\, r^-,\, (q \supset r)^+,\, (p \vee q)^+,\, B^+,\, A \Rightarrow_{\mathbf{C}} q}\,{}^{\mathrm{LC_3}}}{q,\, p^+,\, q^+,\, r^-,\, (q \supset r)^+,\, (p \vee q)^+,\, B^+,\, A \Rightarrow_{\mathbf{C}} p}\,{}^{\mathrm{RC}}}{q,\, p^+,\, r^+,\, (q \supset r)^+,\, B^+,\, A \Rightarrow p \vee q}\,{}^{R_{\vee}{}^+} \qquad \dfrac{}{q,\, p^+,\, r^+,\, (q \supset r)^+,\, B^+,\, A, r \Rightarrow_{\mathbf{C}} r}\,{}^{\mathrm{Ax}}}{\dfrac{\dfrac{q,\, p^+,\, r^+,\, (q \supset r)^+,\, B^+,\, A \Rightarrow p \vee q}{q,\, p^+,\, r^+,\, (q \supset r)^+,\, B^+,\, A, (p \vee q) \Rightarrow_{\mathbf{C}} r}\,{}^{L_{\supset}}}{q,\, p^+,\, r^+,\, (q \supset r)^+,\, B^+,\, A \Rightarrow_{\mathbf{C}} r}\,{}^{\mathrm{LC_3}}}{\dfrac{q,\, p^+,\, r^+,\, (q \supset r)^+,\, B^+,\, A \Rightarrow r}{q,\, p^+,\, (q \supset r)^+,\, B^+,\, A \Rightarrow r}\,{}^{R_{\supset}}}{p^+,\, (q \supset r)^+,\, B^+,\, A \Rightarrow q \supset r}\,{}^{\mathrm{RC}}}{(q \supset r)^+,\, B^+,\, A \Rightarrow p}\,{}^{R_{\vee}{}^+}}{\dfrac{A \Rightarrow B}{\cdot \Rightarrow A \supset B}\,{}^{R_{\supset}}}\,{}^{\mathrm{RC}}$$

The rule

$$\dfrac{(A \vee B)^+,\, B^+,\, \Gamma \Rightarrow A}{\Gamma \Rightarrow A \vee B}\,{}^{R_{\vee}{}^+}$$

can be derived in **MJc** in few steps.

Translation in NC

$$
\begin{array}{rcl}
A & = & (p \vee q) \supset r \\
B & = & p \vee (q \supset r)
\end{array}
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{q, p^1, q^1, r^1, (q \supset r)^1, (p \vee q)^1, B^1, A \vdash q\uparrow}\,\mathrm{id}}{q, p^1, q^1, r^1, (q \supset r)^1, (p \vee q)^1, B^1, A \vdash q\uparrow}\,\mathrm{id}}{q, p^1, q^1, r^1, (q \supset r)^1, (p \vee q)^1, B^1, A \vdash p\uparrow}\,\mathrm{id}}{q, p^1, r^1, (q \supset r)^1, B^1, A \vdash A\downarrow}\,\mathrm{id}}{q, p^1, r^1, (q \supset r)^1, B^1, A \vdash p \vee q\uparrow}\,\mathrm{id}}{q, p^1, r^1, (q \supset r)^1, B^1, A \vdash r\downarrow}\,\mathrm{id}}{q, p^1, r^1, (q \supset r)^1, B^1, A \vdash r\uparrow}\,\mathrm{id}}{p^1, (q \supset r)^1, B^1, A \vdash q \supset r\uparrow}\,\mathrm{id}}{(q \supset r)^1, B^1, A \vdash p\uparrow}\,\mathrm{id}}{\dfrac{A \vdash B\uparrow}{\vdash A \supset B\uparrow}\,\supset r}\,\mathrm{id}
$$

The rule

$$
\dfrac{(A \vee B)^1, B^1, \Gamma \vdash A\uparrow}{\Gamma \vdash A \vee B\uparrow}\,\vee r_1
$$

can be derived in NC in few steps.

Derivations built by `mjc`

Warning
Multisets are represented as sets (formulas are not duplicated) and dots denote empty multisets; $\neg A$ abbreviates $A \supset \bot$.

Formula label: or3

$$(p \wedge (p \supset ((a \supset c) \wedge ((b \supset c) \wedge (a \vee b))))) \supset c$$

Derivation in MJcr

$$\begin{array}{rcl}
A & = & p \supset B \quad = \quad p \supset ((a \supset c) \wedge ((b \supset c) \wedge (a \vee b))) \\
B & = & B_0 \wedge B_1 \quad = \quad (a \supset c) \wedge ((b \supset c) \wedge (a \vee b)) \\
B_0 & = & a \supset c \\
B_1 & = & C_0 \wedge C_1 \quad = \quad (b \supset c) \wedge (a \vee b) \\
C_0 & = & b \supset c \\
C_1 & = & a \vee b \\
H & = & p \wedge A \quad = \quad p \wedge (p \supset ((a \supset c) \wedge ((b \supset c) \wedge (a \vee b))))
\end{array}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\subset p \to p, c \to c}\,\text{Ax}}{p \Rightarrow p, c}\,\text{L}_\bot}{\subset p \to p, c \to c}\,\text{Ax}}{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{p, c \to c \Rightarrow c}\,\text{Ax}}{a, p, C_0 \Rightarrow a, c}\,\text{L}_\bot}{p, C_0, a \to c \Rightarrow a, c}\,\text{Ax}}{p, C_0 \to c \Rightarrow a, c}\,\text{L}_\supset}{p, C_0, C_1 \to c \Rightarrow a, c}\,\text{L}_\wedge}{p, C_1 \to c \Rightarrow a, c}\,\text{L}_\wedge}{p, B_1 \Rightarrow a, c}\,\text{L}_\wedge}{\subset p, B_1 \to c \Rightarrow c}\,\text{L}_\supset}{\subset p, B \to c \Rightarrow c}\,\text{L}_\wedge}{\subset H \to c \Rightarrow c}\,\text{L}_\wedge}{\to H \Rightarrow c}\,\text{L}_\supset}{\to H \supset c}\,\text{R}_\supset}$$

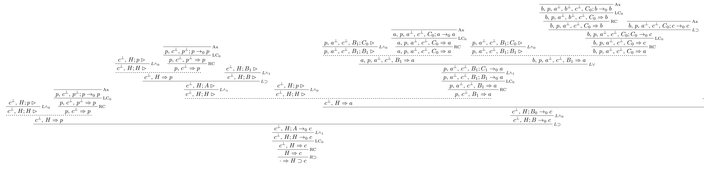
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{p, c \to c \Rightarrow c}\,\text{Ax}}{b, p \Rightarrow b, a, c}\,\text{L}_\bot}{b, p, C_0 \to c \Rightarrow a, c}\,\text{Ax}}{b, p, C_0 \Rightarrow a, c}\,\text{L}_\supset}{b, p, C_0 \to c \Rightarrow a, c}\,\text{Ax}}{b, p, C_0 \Rightarrow a, c}\,\text{L}_\supset}{p, C_0, C_1 \to c \Rightarrow a, c}\,\text{L}_\wedge}{p, C_1 \to c \Rightarrow a, c}\,\text{L}_\wedge}{p, B_1 \Rightarrow a, c}\,\text{L}_\wedge}{\subset p, B_1 \to c \Rightarrow c}\,\text{L}_\supset}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{p, c \to c \Rightarrow c}\,\text{Ax}}{a, p, C_0 \Rightarrow a, c}\,\text{L}_\bot}{p, C_0, a \to c \Rightarrow a, c}\,\text{Ax}}{p, C_0 \to c \Rightarrow a, c}\,\text{L}_\supset}{p, C_0, C_1 \to c \Rightarrow a, c}\,\text{L}_\wedge}{p, C_1 \to c \Rightarrow a, c}\,\text{L}_\wedge}{p, B_1 \Rightarrow a, c}\,\text{L}_\wedge}{\subset p, B_1 \to c \Rightarrow c}\,\text{L}_\supset}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{p, c \to c \Rightarrow c}\,\text{Ax}}{b, p \Rightarrow b, a, c}\,\text{L}_\bot}{b, p, C_0 \to c \Rightarrow a, c}\,\text{Ax}}{b, p, C_0 \Rightarrow a, c}\,\text{L}_\supset}{b, p, C_0 \to c \Rightarrow a, c}\,\text{Ax}}{b, p, C_0 \Rightarrow a, c}\,\text{L}_\supset}{p, C_0, C_1 \to c \Rightarrow a, c}\,\text{L}_\wedge}{p, C_1 \to c \Rightarrow a, c}\,\text{L}_\wedge}{p, B_1 \Rightarrow a, c}\,\text{L}_\wedge}{\subset p, B_1 \to c \Rightarrow c}\,\text{L}_\supset}$$

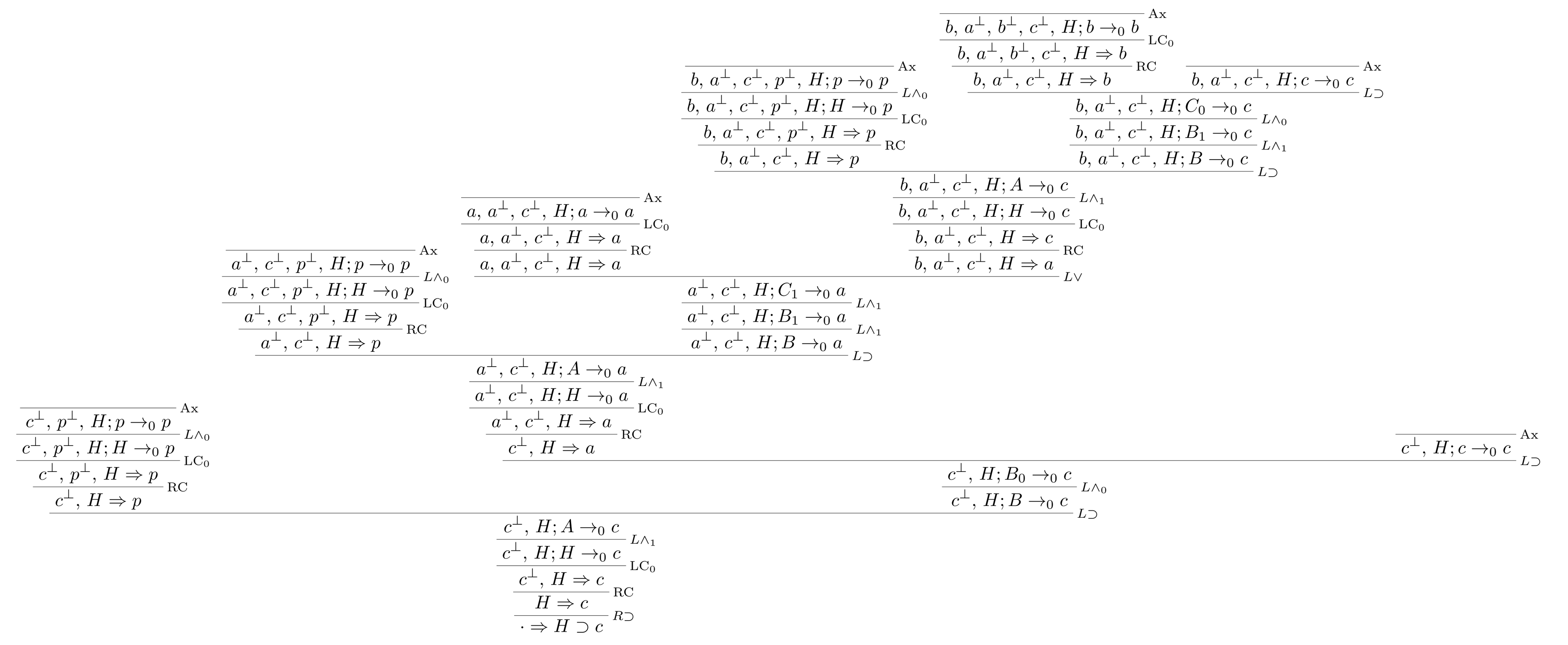
Translation in MJc

$$
\begin{array}{lcl}
A & = & p \supset B \quad = \quad p \supset ((a \supset c) \wedge ((b \supset c) \wedge (a \vee b))) \\
B & = & B_0 \wedge B_1 \quad = \quad (a \supset c) \wedge ((b \supset c) \wedge (a \vee b)) \\
B_0 & = & a \supset c \\
B_1 & = & C_0 \wedge C_1 \quad = \quad (b \supset c) \wedge (a \vee b) \\
C_0 & = & b \supset c \\
C_1 & = & a \vee b \\
H & = & p \wedge A \quad = \quad p \wedge (p \supset ((a \supset c) \wedge ((b \supset c) \wedge (a \vee b))))
\end{array}
$$



Elimination of substitutions

$$\begin{array}{lllll}
A & = & p \supset B & = & p \supset ((a \supset c) \wedge ((b \supset c) \wedge (a \vee b))) \\
B & = & B_0 \wedge B_1 & = & (a \supset c) \wedge ((b \supset c) \wedge (a \vee b)) \\
B_0 & = & a \supset c & & \\
B_1 & = & C_0 \wedge C_1 & = & (b \supset c) \wedge (a \vee b) \\
C_0 & = & b \supset c & & \\
C_1 & = & a \vee b & & \\
H & = & p \wedge A & = & p \wedge (p \supset ((a \supset c) \wedge ((b \supset c) \wedge (a \vee b))))
\end{array}$$



Translation in NC

$$
\begin{array}{lcl}
A & = & p \supset B \quad = \quad p \supset ((a \supset c) \wedge ((b \supset c) \wedge (a \vee b))) \\
B & = & B_0 \wedge B_1 \quad = \quad (a \supset c) \wedge ((b \supset c) \wedge (a \vee b)) \\
B_0 & = & a \supset c \\
B_1 & = & C_0 \wedge C_1 \quad = \quad (b \supset c) \wedge (a \vee b) \\
C_0 & = & b \supset c \\
C_1 & = & a \vee b \\
H & = & p \wedge A \quad = \quad p \wedge (p \supset ((a \supset c) \wedge ((b \supset c) \wedge (a \vee b))))
\end{array}
$$

Derivations built by `mjc`

Warning
Multisets are represented as sets (formulas are not duplicated) and dots denote empty multisets; $\neg A$ abbreviates $A \supset \bot$.

Formula label: or4

$$(((p \wedge q) \vee (p \wedge \neg q)) \vee (\neg p \wedge q)) \vee (\neg p \wedge \neg q)$$

Derivation in MJcr

$$\begin{array}{lcl}
E & = & E_0 \vee D_3 \quad = \quad (((p \wedge q) \vee (p \wedge \neg q)) \vee (\neg p \wedge q)) \vee (\neg p \wedge \neg q) \\
E_0 & = & E_1 \vee D_2 \quad = \quad ((p \wedge q) \vee (p \wedge \neg q)) \vee (\neg p \wedge q) \\
E_1 & = & D_0 \vee D_1 \quad = \quad (p \wedge q) \vee (p \wedge \neg q) \\
D_0 & = & p \wedge q \\
D_1 & = & p \wedge \neg q \\
D_2 & = & \neg p \wedge q \\
D_3 & = & \neg p \wedge \neg q
\end{array}$$

Translation in MJc

$$
\begin{array}{lcll}
E & = & E_0 \vee D_0 & = \quad (((p \wedge q) \vee (p \wedge \neg q)) \vee (\neg p \wedge q)) \vee (\neg p \wedge \neg q) \\
E_0 & = & E_1 \vee D_1 & = \quad ((p \wedge q) \vee (p \wedge \neg q)) \vee (\neg p \wedge q) \\
E_1 & = & D_0 \vee D_1 & = \quad (p \wedge q) \vee (p \wedge \neg q) \\
D_0 & = & p \wedge q & \\
D_1 & = & p \wedge \neg q & \\
D_0 & = & \neg p \wedge q & \\
D_1 & = & \neg p \wedge \neg q &
\end{array}
$$

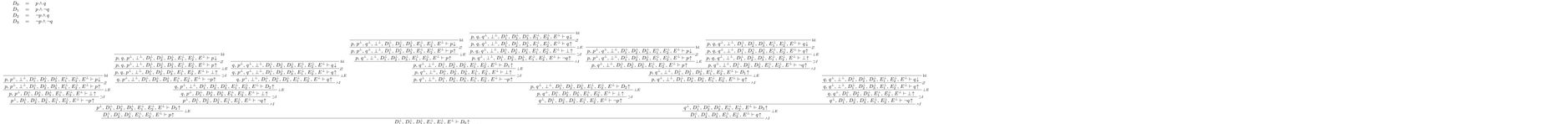
The rule

$$\frac{(A \vee B)^{\perp}, B^{\perp}, \Gamma \Rightarrow A}{\Gamma \Rightarrow A \vee B}\ R_{\vee}{}^{*}$$

can be derived in **MJc** in few steps.

Translation in NC

$$
\begin{array}{lcll}
E & = & E_0 \vee D_0 & = \quad (((p \wedge q) \vee (p \wedge \neg q)) \vee (\neg p \wedge q)) \vee (\neg p \wedge \neg q) \\
E_0 & = & E_1 \vee D_1 & = \quad ((p \wedge q) \vee (p \wedge \neg q)) \vee (\neg p \wedge q) \\
E_1 & = & D_0 \vee D_1 & = \quad (p \wedge q) \vee (p \wedge \neg q) \\
D_0 & = & p \wedge q & \\
D_1 & = & p \wedge \neg q & \\
D_2 & = & \neg p \wedge q & \\
D_3 & = & \neg p \wedge \neg q &
\end{array}
$$



The rule

$$\frac{(A \vee B)^{\uparrow}, B^{\dagger}, \Gamma \vdash \Delta \uparrow}{\Gamma \vdash A \vee B \uparrow}\ {}_{\vee I}$$

can be derived in **NC** in few steps.